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A liquid drop on an air cushion as an analogue of Leidenfrost boiling

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The paper describes the phenomenon of drop suspension above an air-blown porous surface, which is similar to the well-known levitation of a drop evaporating above a hot plate ('spheroidal state' or the Leidenfrost phenomenon). It has been shown that the basis of this similarity is the close analogy between the hydrodynamic mechanisms of drop suspension. Together with the viscous mechanism, the effect of gas- or heat-flow choking under the drop plays an important role here. The latter conditions the threshold character of the above phenomena. A mathematical model of cool- and hot-drop suspension is offered which does not contain any *a priori* assumptions about the drop form and can be applied to the critical range of parameters. An approximation has also been considered in which the bottom of the drop is assumed to be flat, which allows us to carry out an analysis within a wide range of parameters. The simplest version of this approximation is a disk model, where the problems considered are found to be similar. This version allows analytical solution. The model developed has been verified in a 'cool' experiment. The theoretical and experimental data have been shown to be in qualitative (and in some respects in quantitative) agreement.

1. Introduction

The study of the boiling crisis (transition from nucleate to film boiling) is one of the crucial problems of thermophysics. An important step in understanding the mechanisms of the crisis was the description of the hydrodynamical background of boiling on the basis of the analogy between boiling and bubbling (injection of a gas into a liquid through the porous bottom of a vessel) established by Kutateladze (1950, 1979). Such an analogy turns out to be valid not only for processes that occur in a volume, but also for separate liquid drops.

Two regimes can be observed when a drop is placed on a horizontal perforated or porous plate which is blown through from below. With weak injection, the drop spreads over the surface and, being penetrated by bubbles, begins to 'boil' and quickly disperses into small droplets. There exists, however, a critical intensity of the blow rate at which the drop can 'levitate' above the surface without touching it and evaporate slowly (figure 1).

The situation is reminiscent of the well-known change of the boiling regimes for a small portion of a liquid on a heated surface, i.e. nucleate boiling and 'spheroidal state' (Leidenfrost) boiling when the liquid gathers into drops separated from the surface by a thin vapour interlayer (see Kutateladze 1979). In both cases small levitating drops have the form of a ball with a truncated base. The big ones have

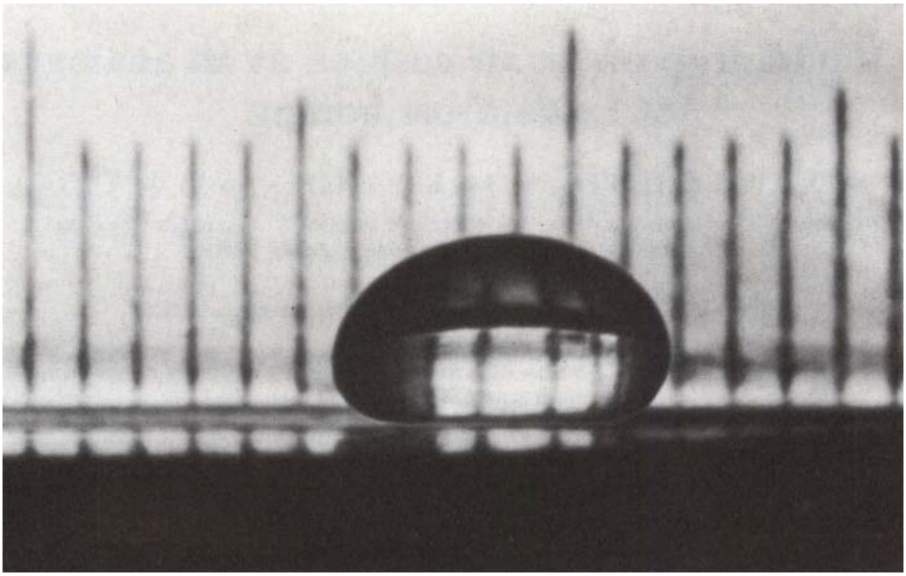

FIGURE 1. Levitation of a drop above a perforated plate. The differential pressure on the plate is close to critical. Divisions on the screen are 1 mm apart. The gap under the drop is ≈ 50 μm.

the form of a 'flat spheroid' of height approximately $2a$, where $a = (\sigma/g\rho_1)^{\frac{1}{2}}$ is the capillary length.

The authors have been unable to find in the literature any description of drop levitation on an air cushion. On the other hand the spheroidal state, which was first described as early as the 18th century (Leidenfrost 1756), is studied in many papers (see Pletenyova & Rebinder 1946; Borishansky & Kutateladze 1947; Wachters, Bonne & Van Nouhuis 1966; Michiyoshi & Makino 1978; Buyevich & Mankevich 1982*a*, *b*). The interest in it is due to both its applications (e.g. heat transfer of a heated surface with a drop-containing flow in nuclear reactors) and its connections with other physical phenomena, such as non-coalescence of contacting drops (Deryagin & Prokhorov 1949). Finally, the visible analogy of the Leidenfrost phenomenon with film boiling seems to be the key to understanding the boiling crisis mechanisms in general (Borishansky & Kutateladze 1947).

The difficulties in experimental and theoretical research on the Leidenfrost phenomenon are due to the influence of secondary factors common to thermal phenomena (air humidity, heat and mass transfer from a drop to an ambient medium, convective and thermocapillary motions in the drop itself and on its surface). 'Cool' experiments (with a drop on an air cushion) are free from the above disadvantages, incomparably simpler to perform and more flexible with respect to parameter variation. This enables us to consider a drop on an air cushion as a convenient model for the spheroidal state. Moreover, one may hope that this phenomenon itself will also have interesting applications, e.g. for non-contact technologies.

2. Qualitative description

Let us consider possible regimes of drop–flow interaction. When the blow rate v_0 is great, the drop can soar above the surface at a height comparable with its radius†

† In practice this is possible only for sufficiently small drops (with $R \ll a$).

R (or $\gg R$). In this case the drop is acted on by the drag force $F \sim \rho_g v_0^2 R^2$, which is determined by the dynamic head. On decreasing v_0, the thickness of the gap under the drop h decreases and at $h \ll R$ the force, still being inertial, is determined by the radial velocity $u \sim v_0 R/h$ and thus $F \sim \rho_g v_0^2 R^4/h^2$. When h attains values of the viscous scale ($\approx \nu/v_0$), the differential pressure over the gap is determined by viscous tensions. Assuming that the blow rate under the drop is fixed, we will get the formula $F \sim \mu_g v_0 R^4/h^3$. Its validity as $h \to 0$ would imply the possibility of suspension of the drop no matter how weak the blow rate. There exists, however, a scale h_* at which the drop begins to 'choke' the gas flow under it, the pressure over the gap becomes equal to the pressure under the porous plate p_1 and the force attains its limiting level $F \sim \Delta p R^2$, where $\Delta p = p_1 - p_0$ and p_0 is the atmospheric pressure. The value of h_* can be estimated by equating the two last expressions for F. Assuming that the plate resistance is linear: $v_0 = (k/\mu_g)\Delta p$, we will have $h_* \sim (kR^2)^{\frac{1}{3}}$. Here k is the plate permeability, the value of the length dimension proportional to the ratio between the square of the characteristic size of the pores and the plate thickness, and therefore is very small.

The drop-suspension condition is the equality of F to the drop weight. Let us stress that the critical parameter that determines the possibility of levitation is the differential pressure on the porous plate Δp and not the blow rate v_0 as it might seem at first glance. It refers, however, to sufficiently smooth surfaces when the characteristic roughness size δ is smaller than h_*. Otherwise, the choking regime cannot be realized and v_0 becomes a critical parameter.

In the case of the Leidenfrost phenomenon a drop is supported above the surface by the vapour overpressure over the gap, determined by the difference ΔT between the surface temperature T_1 and the boiling point T_0 which corresponds to the ambient pressure p_0. An analogue of the blow rate is the vaporization rate $v_{0T} = \lambda \Delta T/hL\rho_v$, where λ is the heat conductivity of the vapour and L is the latent heat of evaporation. As with a drop in an air cushion, two levitation regimes are possible, i.e. inertial, with $F \sim \rho_v v_{0T}^2 R^4/h^2$ (Borishansky & Kutateladze 1947) and viscous, with $F \sim \mu_v v_{0T} R^4/h^3$ (Wachters *et al.* 1966; Baumeister, Hamill & Schoessow 1966). These formulae, obtained on the assumption that the temperature of the drop is constant and equal to T_0, become invalid in the critical range where one should take into account the thermal analogue of choking. As $h \to 0$ the temperature on the drop bottom should tend to T_1 and the pressure over the gap should tend to an appropriate value p_{1T}. Then the heat flux to the drop tends to zero and the force that acts on it attains its limiting level $F \sim \Delta p_T R^2$, where $\Delta p_T = p_{1T} - p_0$. The relation between Δp_T and ΔT is determined by the Clapeyron–Clausius equation, which at $\Delta T \ll T_0$, $\rho_v \ll \rho_l$ reduces to a linear dependence $\Delta T = (T_0/L\rho_v)\Delta p_T$. Equating, as above, the last expressions for F, we find the thermal-choking scale $h_{*T} \approx (lR)^{\frac{1}{2}}$, where $l \sim (\nu_v \lambda T_0/L^2\rho_v)^{\frac{1}{2}}$. For a water drop under normal conditions $h_{*T} \approx 10$ μm at $R \approx 1$ cm.

Thus there exists a distinct analogy between the possible mechanisms of drop suspension in the phenomena under examination. We will obtain below a mathematical expression for this analogy for the viscous regime of levitation (including the choking regime). The characteristic feature of the thermal problem is the explicit dependence of blow rate v_{0T} on the gap thickness h. It leads, in particular, to the fact that the Reynolds number $Re \equiv hv_{0T}/\nu$, which governs the flow regime over the gap, is unambiguously determined by the wall temperature: $Re = \lambda \Delta T/L\mu_v$ and is the same for all suspended drops irrespective of their size. Let us note that for water drops under normal conditions $Re = 1$ only at $\Delta T \approx 1000$ K, so one may almost always assume that $Re \ll 1$.

3. Governing equations

The objective of the theory is to determine the form of a drop and its location with respect to the wall, provided that the drop volume V and Δp (or ΔT) are given. One should determine, in particular, the minimum values of Δp and ΔT at which the thickness of the gap under the drop tends to zero.

The form of the drop is defined by the hydrostatic equation

$$p+\sigma(K_1+K_2)+\rho_1 gh = \text{const.} \tag{1}$$

where $p(r)$ is the gas (vapour) pressure on the drop surface, K_1 and K_2 are the basic curvatures of the surface, $z = h(r)$ is an equation for the surface of the drop over the meridian section. Equation (1) will assume an elegant form, convenient for numerical integration, if we introduce an arclength s and a tangent slope to the contour θ. Then $K_1 = \mathrm{d}\theta/\mathrm{d}s$, $K_2 = \sin\theta/r$. Having chosen as a length unit the capillary constant a and as a pressure unit $\rho_1 ga$ and having made (1) dimensionless, we will reduce (1) to the following system:

$$\theta' = c-p-h-\frac{\sin\theta}{r}, \quad r' = \cos\theta, \quad h' = \sin\theta. \tag{2}$$

Here c is a constant and the prime denotes differentiation with respect to s.

The gas (vapour) flow over the gap can be described by the Stokes equations in terms of the thin-layer approximation (Batchelor 1970):

$$\frac{\partial p}{\partial r} = \mu\frac{\partial^2 u}{\partial r^2}, \quad \frac{\partial p}{\partial z} = 0; \tag{3}$$

$$\frac{1}{r}\frac{\partial ru}{\partial r}+\frac{\partial v}{\partial z} = 0. \tag{4}$$

The boundary conditions for (3), (4) and the further derivation of equations will be discussed individually for the cases under study.

3.1. *A drop on an air cushion*

For the surface of a pure-liquid drop we should formulate the continuity condition of tangential stresses. However, due to the concentration of surfactants under normal technical conditions, the no-slip condition seems to be more suitable (and much simpler):

$$v = u = 0 \quad \text{at} \quad z = h(r). \tag{5}$$

The simplest formulation of the boundary conditions for a porous surface is to give a uniform blow rate: $v = v_0$, $u = 0$ at $z = 0$. However, in the critical range of parameters it leads to a contradiction, and so it is necessary to solve the conjugate problem of a flow in a porous plate. Nevertheless, when its thickness is much smaller than the drop radius, we can assume that the blow rate is determined by the local differential pressure and neglect its radial component:

$$v = \frac{k}{\mu_g}[p_1-p(r)], \quad u = 0 \quad \text{at} \quad z = 0. \tag{6}$$

From (3) and (5) the representation for the radial velocity over the gap is: $u = -\frac{1}{2\mu_g}z[h(r)-z]\,\mathrm{d}p/\mathrm{d}r$. Substituting it into the continuity equation (4), integrating the latter from 0 to h and equating the expression obtained for $v(r,0)$ to (6), we obtain an equation for the pressure distribution over the gap:

$$\frac{\mathrm{d}}{\mathrm{d}r}\left(rh^3\frac{\mathrm{d}p}{\mathrm{d}r}\right)+12kr(p_1-p)=0. \tag{7}$$

To solve the problem completely, we need equations describing the flow around a drop. However, when the drops are not too small, the pressure varies substantially only over the gap and in the neighbouring narrow region; beyond this it is practically constant. Therefore the objective can be achieved with the help of an appropriate extrapolation of (7) from the gap region. The easiest way to do this is to formally extrapolate (7) to the entire contour of the drop after the substitution of $\mathrm{d}/\mathrm{d}r$ by $\mathrm{d}/\mathrm{d}s$ (we may also substitute s for r everywhere in (7)). Let us also make (7) dimensionless with the help of the introduced scales of length a and pressure $\rho_1 ga$ and reduce it to a system of first-order equations:

$$p'=-\frac{\gamma}{rh^3}j,\quad j'=r(p_1-p)\qquad\left(\gamma=\frac{12k}{a}\right). \tag{8}$$

The above extrapolation possesses the necessary qualities, since system (8) is in practice similar to (7) over the gap where $r\approx s$ and the pressure rapidly becomes constant beyond it owing to the smallness of the parameter γ.

3.2. *Leidenfrost drop*

In formulating the thermal problem we will use the simplest assumptions. For the vapour flow over the gap the following boundary conditions are assumed:

$$v=u=0\quad\text{at}\quad z=0; \tag{9}$$

$$v=-\frac{\lambda}{hL\rho_{\mathrm{v}}}[T_1-T(r)],\quad u=0\quad\text{at}\quad z=h(r). \tag{10}$$

The parameters λ and ρ_{v} may be assumed as being given at a temperature average of T_0 and T_1 (Wachters *et al.* 1966). The no-slip condition $u=0$ for the drop surface should be treated carefully owing to possible thermocapillary phenomena. Nevertheless, this condition is generally used in papers on the Leidenfrost phenomenon. The first condition in (10) requires special comments. First, it has been obtained on the assumption of a z-linear temperature profile over the gap. This assumption is justified by the *a posteriori* estimation of the convective terms in the thermal-conductivity equation. Secondly, the temperature of the drop surface T is usually assumed in (10) as being constant and equal to T_0. This assumption cannot, however, be used in the critical range of parameters, as was pointed out first by Buyevich & Mankevich (1982a, b).†)

Let us now consider the dependence of T on the gap pressure p described by the Clapeyron–Clausius equation. We will use its linear approximation, which is valid for $T-T_0\ll T_0$, and assume that $\rho_{\mathrm{v}}\ll\rho_1$. Then

$$T-T_0=A(p-p_0),\quad A=\frac{T_0}{L\rho_{\mathrm{v}}}.$$

† The analysis in these papers has been carried out in terms of the disk model (see §5.2). Unfortunately, the authors made an error in deriving the equations which led to an inaccuracy of the results and affected some qualitative conclusions.

Let us introduce the value $p_{1T} = p_0 + \Delta T/A$, which coincides for small ΔT with the saturation pressure at a temperature T_1. Then we can write $T_1 - T = A(p_{1T} - p)$ and condition (10) will assume a form similar to (6). Now, analogously to the derivation of (7) and (8), we obtain an equation for the pressure over the gap:

$$\frac{\mathrm{d}}{\mathrm{d}r}\left(rh^3\frac{\mathrm{d}p}{\mathrm{d}r}\right) + \frac{12l^2 r}{h}(p_{1T} - p) = 0, \quad l^2 = \frac{\nu_\mathrm{v}\lambda T_0}{L_\mathrm{f}^2 \rho_\mathrm{v}}, \tag{11}$$

and an extrapolation system

$$p' = -\frac{\gamma_T}{rh^3}j, \quad j' = \frac{r}{h}(p_{1T} - p) \quad \left(\gamma_T = \frac{12l^2}{a^2}\right). \tag{12}$$

Let us note that for water drops under normal conditions the value of the parameter γ_T is only 1.1×10^{-10}.

The difference between (7), (8) and (11), (12) arises because the evaporation rate, unlike the blow rate, depends explicitly on h. Therefore the derived equations are not similar to each other, though the analogy between them is sufficiently close.

3.3. *Problem formulation*

Equations (2) and (8) (or (12)) form a closed system. We can integrate them having specified the initial conditions at the lowest point on the drop axis,

$$\theta = r = j = 0, \quad h = h_\mathrm{a}, \quad p = p_\mathrm{a} \quad \text{at} \quad s = 0,$$

up to the value $s = s_1$, which, together with one of the parameters c, h_a, p_a, is determined as an eigenvalue from the drop-contour closure conditions:

$$\theta(s_1) = \pi, \quad r(s_1) = 0.$$

The two other parameters correspond to the given experimental quantities, i.e. to the ambient pressure p_0 and the drop volume V.

The calculations showed that it is useful to fix the parameters h_a and p_a. In so doing, the eigenvalue c and the corresponding value s_1 were determined by the bisection method.

4. Numerical results

Figure 2 shows an example of a calculation which illustrates the characteristics of the drop structure and pressure distribution on the drop surface. The calculation refers to a drop on an air cushion and the value of γ corresponds to our experimental conditions. All the values given are dimensional (a is chosen as being equal to 2.5 mm). One should note the table-like pressure profile over the gap, which is due to the effect of flow choking.

Figure 3 presents the minimum gap thickness h_m and the gap thickness on the axis, h_a *versus* Δp for drops of different sizes. The maximum radius R of a drop is chosen as its geometrical characteristic. The value of $\Delta p = \Delta p_\mathrm{m}$ for which $h_\mathrm{m} = 0$ is the critical differential pressure. It is interesting that the minimum of h is attained in the drop periphery and its bottom is concave (as in figure 2) only at sufficiently large values of Δp. Otherwise, $h_\mathrm{m} = h_\mathrm{a}$, the bottom is convex and remains the same until the 'landing' of the drop on the surface at Δp_m.

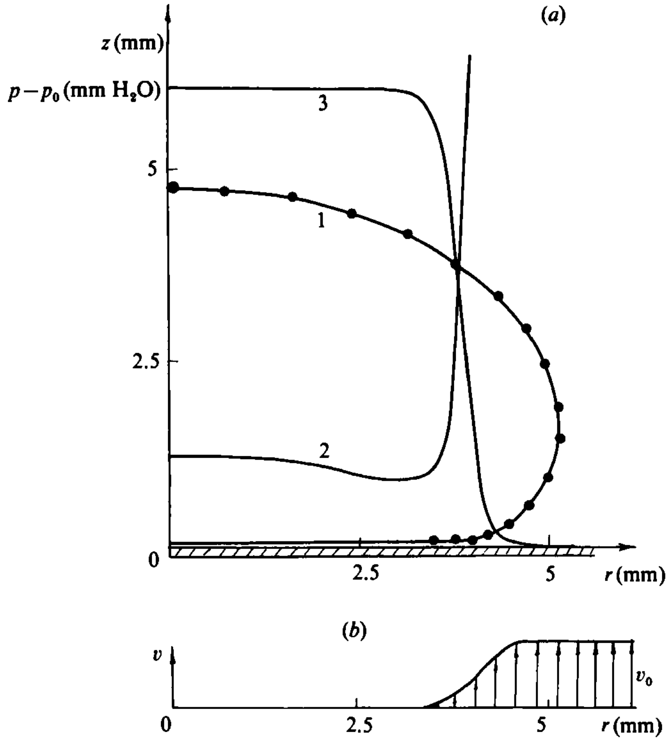

FIGURE 2. (*a*) Drop form (curve 1: ——, calculation; ●, experiment), profile of drop bottom (curve 2: ——, calculation, along the vertical a 200-fold increase) and pressure distribution around drop contour (curve 3: ——, calculation). (*b*) The distribution of the injection rate on the porous surface is shown schematically; $\gamma = 10^{-3}$, $\Delta p = 2.45\rho_1 ga$.

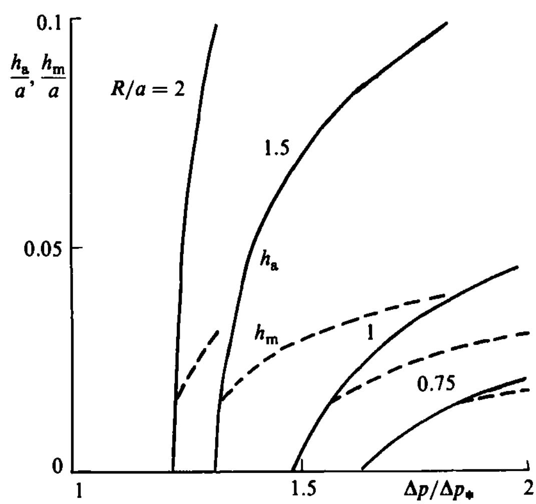

FIGURE 3. Gap thickness on the drop axis h_a and minimum gap thickness h_m *vs* differential pressure on a porous plate for drops of various sizes on an air cushion; $\Delta p_* = 2\rho_1 ga$.

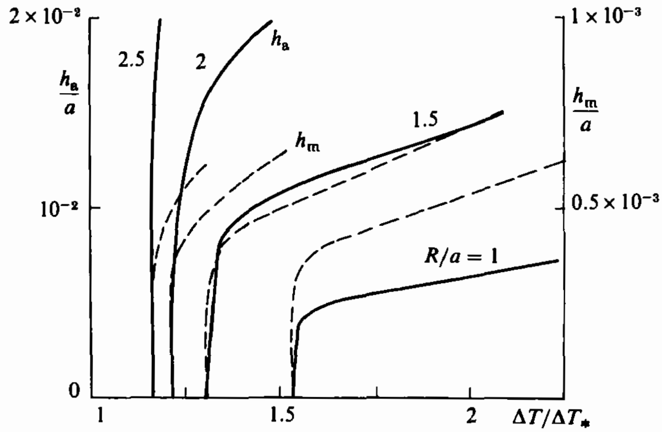

FIGURE 4. Gap thickness on the drop axis h_a and minimum gap thickness h_m *vs* surface temperature for Leidenfrost drops; $\Delta T_* = 2\rho_l g a T_0 / L\rho_v$.

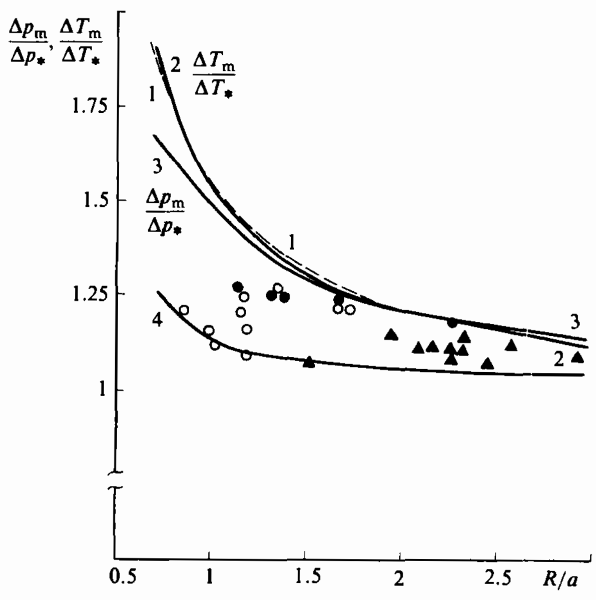

FIGURE 5. Lowest-possible pressure Δp_m and temperature ΔT_m differences for drop levitation *vs* drop size. Curve 1, 'disk' model; curves 2, 3, exact solution, for $\gamma_T = 1.1 \times 10^{-10}$ and $\gamma = 10^{-3}$ respectively; curve 4, flat-bottom approximation with allowance for the lateral surface of the drop, for $\gamma = 10^{-3}$. Experiment: minimum differential pressure for drops with ●, $a = 2.48$; ○, 2.60; ▲, 2.70 mm. The permeability of the perforated plate $k = 0.24 \times 10^{-6}$ m.

The bottom of the Leidenfrost drop has a different structure (figure 4): it is always concave and is flattened only if the temperature tends to the critical value ($\Delta T \to \Delta T_m$). At values of ΔT which are close to ΔT_m, h_a and h_m vary extremely sharply, which means that there exists a certain interval of gap thicknesses where Leidenfrost drops cannot be observed in practice.

Nevertheless, despite the differences in the structure of the bottom, the dependence of the critical parameters on the drop size (figure 5) is similar. It is noteworthy that, with increasing R, they decrease, which means that it is easier to suspend a drop, the larger its size.† The quantities

$$\Delta p_* = 2\rho_1 g a, \quad \Delta T_* = \frac{2\rho_1 g a T_0}{L\rho_1}, \tag{13}$$

with which Δp and ΔT are scaled in figures 3–5 represent the limiting values of the critical parameters as $R \to \infty$. For water drops under normal conditions $\Delta T_* \approx 0.013$ K. This result, which means that the drop can be suspended at a negligible surface superheat,‡ seems to be rather unexpected at first sight, but is qualitatively confirmed by experiment (Gossar 1895; Wachters *et al.* 1966).

5. Flat-bottom approximation

The solution of the problems thus formulated (as the nonlinear eigenvalue problems) is rather time consuming. Therefore the subsequent analysis will be carried out in terms of an approximation in which the nonlinear part of the problem related to the determination of the drop form will be separated from the general scheme. This can be achieved with the help of the assumptions, normal in papers on the Leidenfrost phenomenon, that the drop bottom is flat and in calculating its lateral surface the pressure may be assumed to be constant. In terms of this approximation, the drop form is unambiguously determined by its volume and only the gap thickness h is a function of Δp or ΔT.

5.1. *Drop form*

The form of a drop with a flat bottom can be calculated in a more convenient way, if we begin with its top. Formally this can be achieved by the substitutions $s \to -s$, $\theta \to \pi - \theta$, $h \to -y$ in system (2), where we now assume that $p = \text{const}$. This reduces (2) to the form

$$\theta' = c + y - \frac{\sin\theta}{r}, \quad r' = \cos\theta, \quad y' = \sin\theta. \tag{14}$$

We can integrate system (14), having specified the initial conditions $\theta = r = y = 0$ at $s = 0$ up to the value $s = s_0$ at which the tangent to the contour will be horizontal again: $\theta(s_0) = \pi$.

Figures 6(*a*, *b*) show results of the calculations of the drop volume V, its height H, base radius $R_0 = r(s_0)$, effective height $H_e = V/\pi R_0^2$ and surface curvature on the line of conjugation with the flat bottom, $K_0 = \theta'(s_0)$ (the two last values are related by the equality $H_e = a^2 K_0$) as functions of the maximum drop radius R (see also Wachters *et al.* 1966). The asymptotic form of these functions at $R \ll a$ can be found if we use the intuitive notion that the form of a small suspended drop is a ball with a truncated base of radius $R_0 \ll R$. Then $H \approx 2R$, $V \approx \frac{4}{3}\pi R^3$ and the force balance will give $\rho_1 g V = \pi R_0^2 \Delta p_c$, where $\Delta p_c = 2\sigma/R$ is the pressure jump at the exit from the gap, which is the same as the capillary jump on the lateral surface. Thence $R_0 \approx \sqrt{\frac{2}{3}} R^2/a$, $H_e \approx 2a^2/R$. As $R \to \infty$ we have H, $H_e \to 2a$, $R_0 \sim R$, $V \sim 2a\pi R_0^2$. Control calculations carried out in terms of the full formulation of §3 have shown that the form of the lateral surface actually depends only slightly on Δp and ΔT and can be well described by a flat-bottom approximation.

† Here we have not taken into account the possible instabilities of the surface which are more typical of large drops and make their suspension difficult (see §§6.2, 6.3).

‡ This conclusion, like the previous one, does not take into account the instability of the drops and also the roughness of the wall (see §5.2).

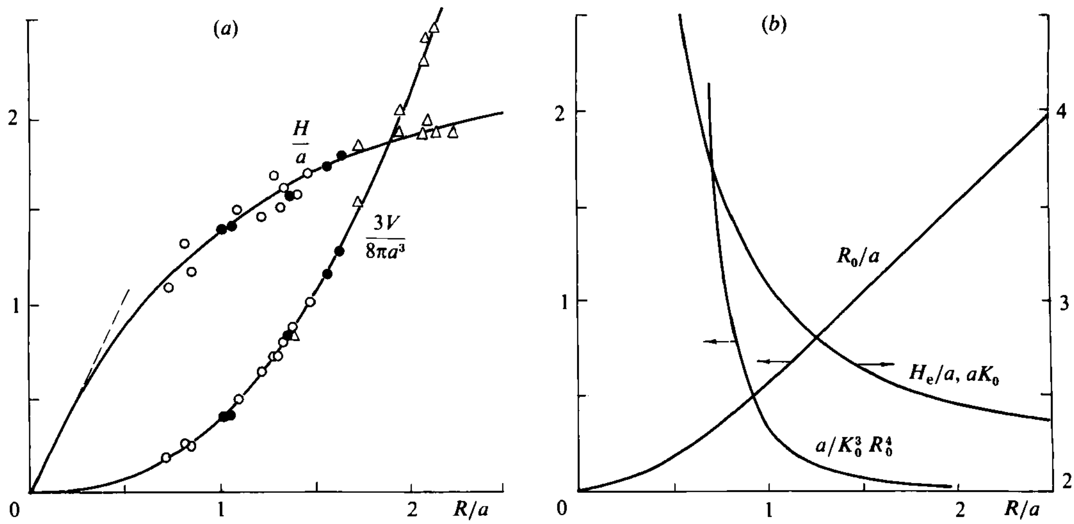

FIGURE 6. (a, b) Mutual dependence between geometrical parameters of levitating drops (§§5.1, 5.3). Experimental data: ○, $a = 2.70$ (pure water); △, 2.65, ●, 2.48 mm (water with organic additives).

5.2. *Disk model*

We will come to the simplest drop-suspension model, setting $h = \text{const.}$ in (7), (11) and taking as boundary conditions for these equations (together with the axial-symmetry condition) that the pressure becomes equal to atmospheric immediately beyond the gap:

$$\frac{\mathrm{d}p}{\mathrm{d}r} = 0 \quad \text{at} \quad r = 0, \qquad p = p_0 \quad \text{at} \quad r \geqslant R_0. \tag{15}$$

Actually the drop is substituted by a flat disk of radius R_0 which depends on the drop volume. The substitution of the variables $x = r/R_0$, $\phi = (p_1 - p)/\Delta p$ reduces problems (7), (15) and (11), (15) to the identical form

$$(x\phi')' = \beta x \phi; \quad \phi'(0) = 0, \; \phi(1) = 1. \tag{16}$$

Thus in terms of this model the 'cool' and thermal problems turn out to be similar. The similarity parameter β is equal to

$$\beta = \left(\frac{h_*}{h}\right)^3, \quad \beta \equiv \beta_T = \left(\frac{h_{*T}}{h}\right)^4, \quad h_* = (12kR_0^2)^{\frac{1}{3}}, \quad h_{*T} = (2\sqrt{3}\, lR_0)^{\frac{1}{4}}, \tag{17}$$

respectively, and the values h_* and h_{*T} are the choking scales for both problems.

The solution of (16) is described by modified Bessel's functions:

$$\phi(x) = I_0(\beta^{\frac{1}{2}}x)/I_0(\beta^{\frac{1}{2}}).$$

The force that acts on the drop and the gas flow from the gap is defined by the formulae

$$\left.\begin{aligned} F = \pi R_0^2 \Delta p f(\beta), \quad Q = \pi R_0^2 v_0 \Phi(\beta), \\ v_0 = \frac{k}{\mu_g}\Delta p, \quad \Phi = \frac{2I_1(\beta^{\frac{1}{2}})}{\beta^{\frac{1}{2}} I_0(\beta^{\frac{1}{2}})}, \quad f(\beta) = 1 - \Phi(\beta). \end{aligned}\right\} \tag{18}$$

The dimensionless force f, together with its asymptotics, is given in figure 7(a) as a function of β and in figure 7(b) as a function of the gap thickness h/h_* (for the

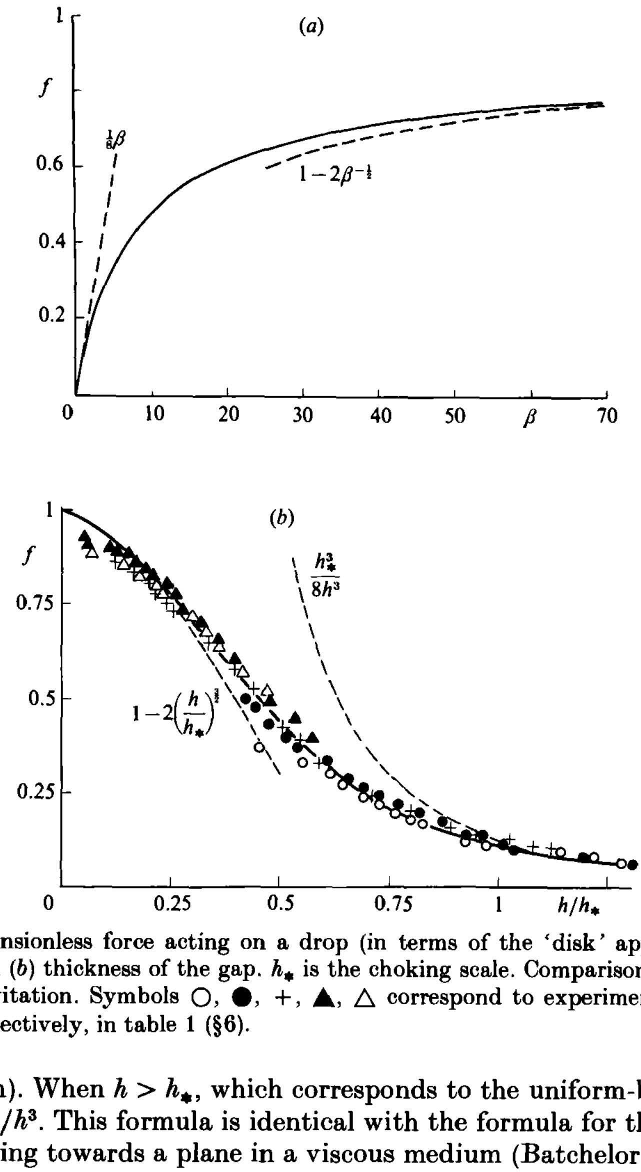

FIGURE 7. Dimensionless force acting on a drop (in terms of the 'disk' approximation) *vs* (*a*) parameter β and (*b*) thickness of the gap. h_* is the choking scale. Comparison with experimental data on disk levitation. Symbols ○, ●, +, ▲, △ correspond to experimental series numbers 1, 2, 3, 4, 5, respectively, in table 1 (§6).

'cool' problem). When $h > h_*$, which corresponds to the uniform-blow assumption, $F = \frac{3}{2}\pi\mu_g v_0 R^4/h^3$. This formula is identical with the formula for the force that acts on a disk moving towards a plane in a viscous medium (Batchelor 1970). At $h < h_*$ we should take into account the choking effect. As $h \to 0$ we have $Q \to 0$, $F \to \pi R_0^2 \Delta p$.

When applying this to the thermal problem, in (18) we should set

$$\Delta p \to \Delta p_T = \frac{\rho_v L \Delta T}{T_0}, \quad v_0 \to v_{0T} = \frac{\lambda \Delta T}{\rho_v L h}. \tag{19}$$

In this case Q has the sense of the full vapour flow from the drop bottom, which is connected with the heat flux flowing into the drop: $q = \rho_v LQ$. The dependence of the dimensionless flux on the gap thickness is given in figure 8. The case $h > h_{*T}$ corresponds to the applicability of the assumption about the constant temperature of the drop. In this case

$$F = \frac{3\pi}{2}\frac{\nu_v \lambda \Delta T R_0^4}{\rho_v L h^4}, \quad q = \pi R_0^2 \frac{\lambda \Delta T}{h},$$

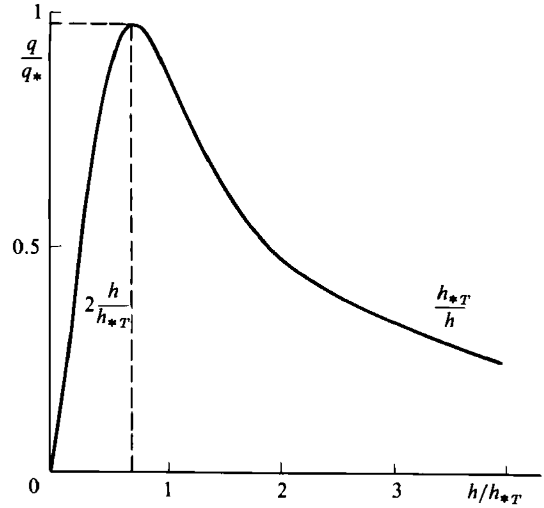

FIGURE 8. Heat flux to the drop bottom as a function of gap thickness. Asymptotics are given for small and large h/h_{*T}, where h_{*T} is the thermal-choking scale; $q_* = \pi R_0^2 \lambda \Delta T / h_{*T}$.

which corresponds to the well-known formulae (Wachters *et al.* 1966). As $h \to 0$ we have

$$F \to \pi R_0^2 \Delta p_T, \quad q = \pi R_0^2 \frac{\lambda \Delta T}{h_{*T}^2} h.$$

It is noteworthy that the heat flux tends to zero as both $h \to \infty$ and $h \to 0$. At the same time, the force F (as with a drop on an air cushion) tends to a certain finite limit, which conditions the critical character of the phenomena under study.

Setting F from (18), (19) equal to the drop weight $\rho_1 g \pi R_0^2 H_e$ we obtain the relations

$$\Delta p f(\beta) = \Delta p_m, \quad \Delta T f(\beta_T) = \Delta T_m, \quad \Delta p_m = \rho_1 g H_e, \quad \Delta T_m = \frac{T_0}{\rho_1 L} \Delta p_m, \tag{20}$$

which, together with (17), determine the dependence of the height of suspension h of a drop of given volume on the pressure or temperature difference. Since $f \to 1$ as $h \to 0$, the values Δp_m and ΔT_m from (20) are critical parameters in terms of this approximation. Their dependence on the drop size is defined by the similarity relations

$$\frac{\Delta p_m}{\Delta p_*} = \frac{\Delta T_m}{\Delta T_*} = \frac{H_e}{2a} \tag{21}$$

and is given in figure 5. The values Δp_* and ΔT_* are defined by (13).

In experiments the height of drop suspension is limited to the roughness of the surface δ. The appropriate critical parameters Δp_δ and ΔT_δ can be estimated from the above relations by substituting δ for h. The dependence of ΔT_δ on δ is given in figure 9. Let us note that at $\delta > h_{*T}$ there exists a very strong dependence $\Delta T_\delta \sim \delta^4$. This is the reason why the Leidenfrost phenomenon can usually be observed at sufficiently great superheats of the surface, though the theoretical limit of the Leidenfrost temperature ΔT_m is very small.

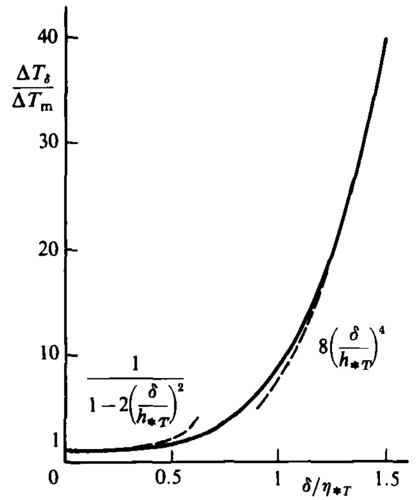

FIGURE 9. Leidenfrost temperature *vs* surface roughness δ. Asymptotic forms of this function are shown.

5.3. *The influence of the lateral surface*

In the case of sufficiently small drops the disk model is inadequate because the contribution of the pressure to the lateral surface to the force F becomes substantial. To a lesser extent this concerns the thermal problem, since the evaporation rate (as distinguished from the injection rate) beyond the gap rapidly tends to zero.†

In this section we will calculate F for drops on an air cushion, assuming that the function $h(r)$ in (7) is defined by the formulae

$$h = h_a \quad \text{at} \quad r \leqslant R_0, \qquad h = h_a + \tfrac{1}{2}K_0(r-R_0)^2 \quad \text{at} \quad r \geqslant R_0. \tag{22}$$

The substitution of the true dependence $h(r)$ by a branch of a parabola that is the asymptotic form of h as $r \to R_0$ is justified by the fact that the pressure beyond the gap rapidly becomes constant. The boundary conditions for (7) are now the axial symmetry and the asymptotical tending of p to the value p_0 as $r \to \infty$.

The solution of this problem is determined by two dimensionless parameters in the context of which it is convenient to choose h_a/h_* and the complex $\Lambda = 96k/K_0^3 R_0^4$, proportional to the product of the purely physical parameter k/a and the geometrical one $\xi = a/K_0^3 R_0^4$, which is determined by the relative drop size alone. The dependence of ξ on R/a is given in figure 6(*b*). At $R > 2a$, ξ rapidly tends to zero (like $\frac{1}{8}(a/R)^4$) and, at $R < \frac{1}{2}a$, ξ tends to infinity (like $\frac{9}{32}(a/R)^5$). Figure 10 shows the force that acts on a drop as a function of gap thickness for various values of Λ. The curve $\Lambda = 0$ (large drops or small permeability of the porous plate) is the same as the curve in figure 7(*b*) which has been obtained in terms of the disk model.

The complex Λ is convenient because it is the only governing parameter in the limiting case $h_a \to 0$. The dependence of the limiting values of the force f_* on Λ is given in figure 11. The difference $f_* - 1$ characterizes the relative contribution of the lateral surface to the interaction with the flow. Using this curve, we can determine the critical differential pressure Δp_m as a function of the drop size. An example of such a calculation for a specific value of the parameter k/a is compared in figure 5 with the

† Nevertheless, the contribution of the lateral surface to the total vapour flux may be substantial (Wachters *et al.* 1966).

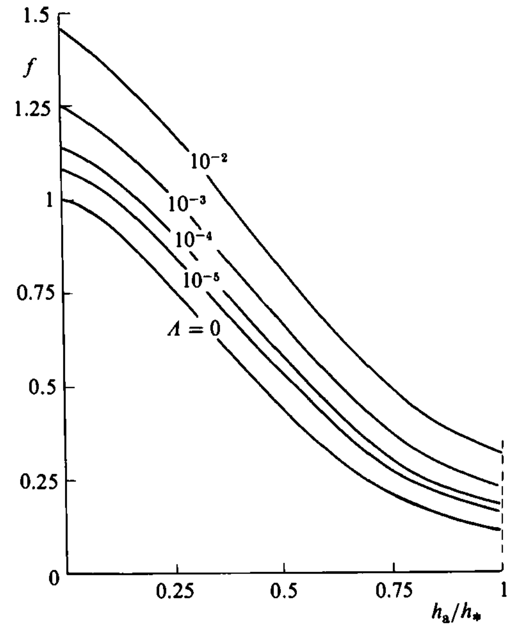

FIGURE 10. Force acting on a drop with a flat bottom with allowance for its lateral surface, $A = 96k/K_0^3 R_0^4$.

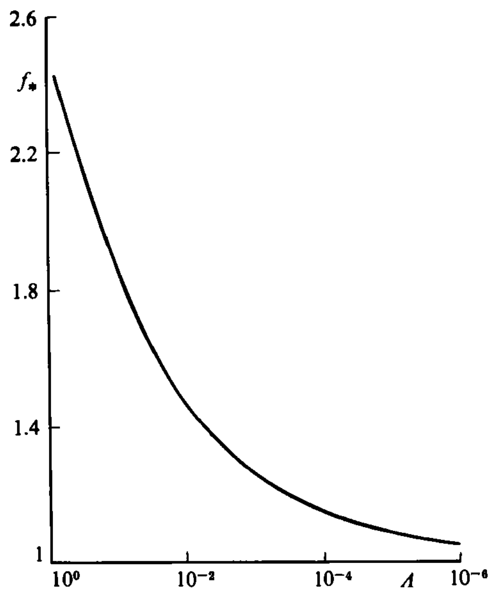

FIGURE 11. Limiting values of the force when the thickness of the gap under the drop is zero.

results obtained with the full formulation of the problem and those obtained in terms of the disk model. We should note that the closeness of these results is coincidental and connected with the mutual compensation of two factors that have not been taken into account in the disk model, i.e. the curvature of the drop bottom (convex as $h \rightarrow 0$) and the influence of the lateral surface.

Series number... Parameter	1	2	3	4	5
$D = 2R_0$ (mm)	15	47	15	22	15
m (g)	6	36	0.92	2.75	0.92
k (m)	0.86×10^{-9}	0.86×10^{-9}	0.63×10^{-7}	0.63×10^{-7}	0.24×10^{-6}
h_* (mm)	0.08	0.18	0.35	0.45	0.54
Re	0.1/2.5	0.1/3	0.1/30	0.4/10	2/35
Re_1	0.02/0.15	0.01/0.1	7/40	10/20	25/50

TABLE 1. Parameters of experimental series on disk suspension

6. Experiments and discussion

6.1. *Experiments with disks*

As we have already mentioned, a drop on an air cushion may be considered as a very convenient 'cool' model of the Leidenfrost phenomenon, which allows us to avoid a number of complicating factors. We may go further and free ourselves of all the factors connected with the variability of the drop form and the motion of the liquid inside the drop, and thereby single out the hydrodynamic-suspension mechanism. To this effect, we should study the levitation of 'solidified' drops – in the simplest case, of disks.

The results of such experiments, which we have performed to verify the dynamic part of the theory that we have developed, are given in figure 7(*b*), which shows the dependence of the dimensionless force $f = mg/\pi R_0^2 \Delta p$ (m is the mass of the disk) on the gap thickness h/h_*. The value h was determined from the measured electrical capacity of the gap and varied within the range 10 μm–1 mm. Table 1 summarizes the parameters of the experimental series, including the range of Reynolds numbers calculated from v_0 and $h(Re)$ and the mean velocity in the pores (holes) of the plate and their characteristic size (Re_1). The first two data series refer to a plate of porous steel, the other three to perforated smooth plates.

As is seen from figure 7(*b*) and table 1, the data from the very wide range of parameters are well generalized and completely agree with the threory that takes into account the choking effect. It is noteworthy that the agreement with experiment can be observed far beyond the limits where the assumptions which underlie this linear theory (such as $Re, Re_1 \ll 1$) are valid. This is because the choking effect forces the injection out to the periphery of the drop and the greater part of its base is occupied by the 'shadow' where the blow rates are small.

6.2. *Geometry of suspended drops: self-sustained oscillations*

Experiments with drops were performed on a smooth plate with $k = 0.24 \times 10^{-6}$ m. We studied water drops which were commercially pure and with the addition of carbomethylcellulose, which increases the stability of the surface. Tables 2 and 3 and figures 2 and 6(*a*) show some data on the drop form which have been obtained from photographs of drops taken in profile (as in figure 1). These data confirm the fact that the form of the lateral surface of stationary drops depends on the injection intensity only slightly and is satisfactorily described in terms of the flat-bottom approximation. The data on the dependence between R and V agree extremely well. Let us note that within the wide interval of sizes $\frac{1}{2}a < R < 3a$ the curve $V(R)$ can be described by the empirical formula $V = 3.10(aR^5)^{\frac{1}{2}}$, which can be

	Series 1 ($a = 2.7$ mm)										
V (mm^3)	25	40	40	81	104	125	129	131	135	161	188
$2R$ (mm)	3.90	4.44	4.62	5.92	6.58	6.97	7.21	7.34	7.48	7.90	8.41
H (mm)	2.90	3.56	3.14	4.01	4.18	4.48	4.31	4.24	4.21	4.48	4.62
Δp (mm H_2O)	10.4	8.2	7.8	8.4	7.4	7.4	7.2	7.0	7.4	7.0	7.0
C_E (pF)	0.77	1.02	0.77	1.43	2.04	2.01	2.04	2.16	2.01	2.16	2.43
h_a (μm)	58	66	73	106	95	114	126	122	141	149	157
Δp (mm H_2O) (theory)	8.5	8.2	8.3	9.2	8.0	8.5	9.0	8.7	9.5	7.9	9.4
	Series 2 ($a = 2.48$ mm)										
V (mm^3)	19	48	52	90	107	145	160				
$2R$ (mm)	3.50	5.04	5.26	6.46	6.89	7.74	8.12				
H (mm)	2.63	3.38	3.46	3.69	3.86	4.21	4.35				
Δp (mm H_2O)	6.3	7.8	9.4	8.6	6.8	6.6	7.1				
C_E (pF)	0.66	1.6	1.2	1.5	3.0	3.3	2.4				
h_a (μm)	52	65	95	137	72	90	152				
Δp (mm H_2O) (theory)	7.4	7.0	8.4	9.3	6.4	6.6	8.4				

TABLE 2. Geometrical parameters of suspended drops. Differential pressure on a perforated plate *versus* gap thickness.

	Series 1 ($a = 2.44$ mm)							
V (mm^3)	57.0	58.1	86.9	103	167	363		
$2R$ (mm)	5.58	5.60	6.33	6.71	8.17	11.0		
H (mm)	3.33	3.35	3.79	4.08	4.33	5.17		
Δp_m (mm H_2O)	6.3	6.0	6.2	6.2	6.3	6.0		
	Series 2 ($a = 2.60$ mm)							
V (mm^3)	32.7	44.8	52.5	76.0	80.4	116	199	216
$2R$ (mm)	4.33	4.92	5.21	5.96	6.08	7.08	8.58	8.92
H (mm)	3.00	3.33	3.50	3.83	3.83	4.02	4.83	4.83
Δp_m (mm H_2O)	6.2	6.0	5.8	6.5	5.7	6.6	6.4	6.3
	Series 3 ($a = 2.65$ mm)							
V (mm^3)	124	232	308	347	367	375	402	
$2R$ (mm)	7.29	9.21	10.29	10.96	11.11	11.29	11.79	
H (mm)	4.07	4.79	5.00	4.93	5.14	5.00	4.93	
Δp_m (mm H_2O)	5.8	6.2	6.0	5.9	6.0	6.2	5.8	

TABLE 3. Geometrical parameters of drops and minimum differential pressure

used for the precise determination of the volume of suspended drops from diameter measurements.

The data-spread in figure 6(a) with respect to the drop height H is due to the existence of self-sustained oscillations of a drop (figure 12), which always arise when a certain critical injection intensity is attained. These oscillations are analogous to the oscillations of Leidenfrost drops which were first described by Stark (1898) and studied by Wachters *et al.* (1966). The latter showed that the frequency of these

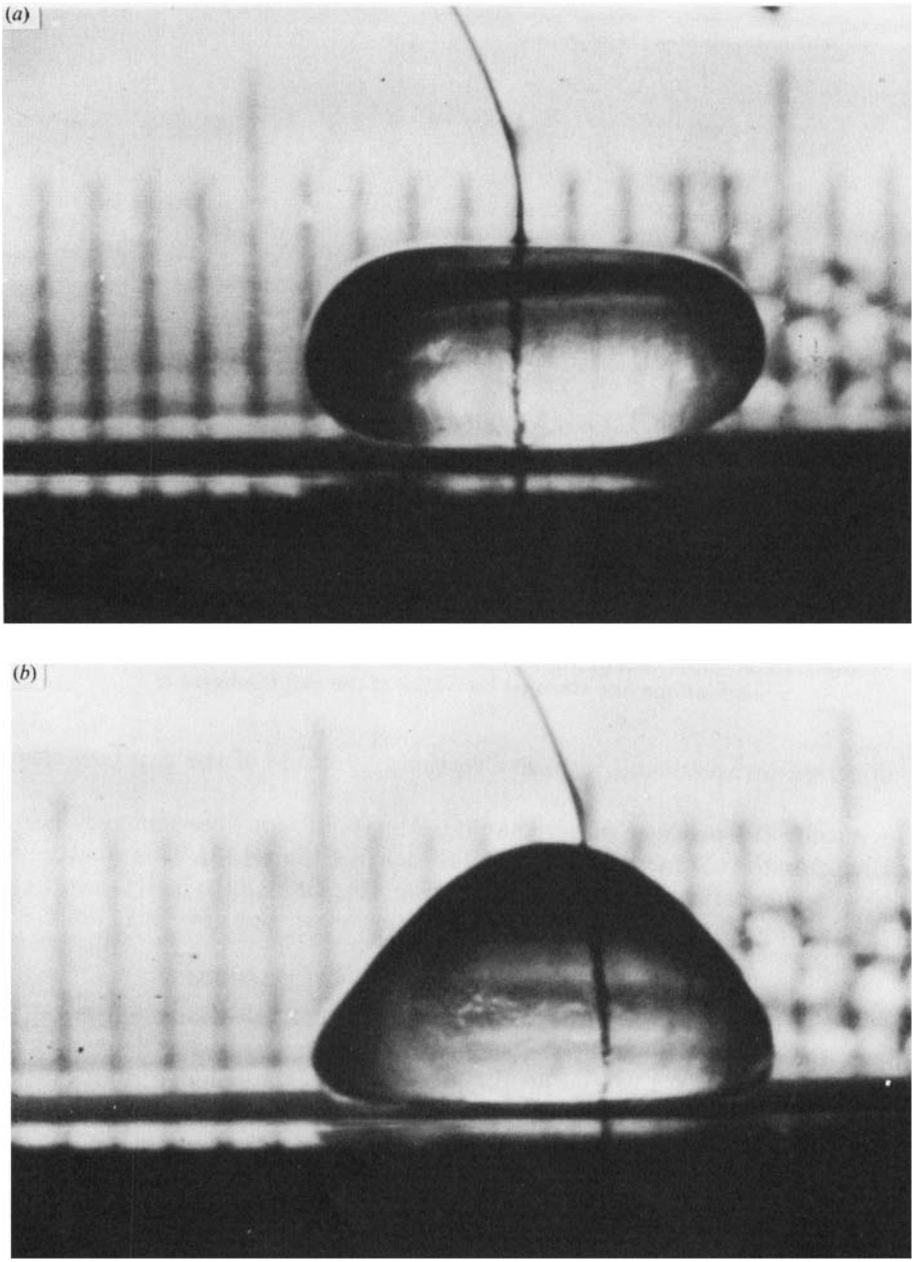

FIGURE 12. (a, b) Opposite phases of self-sustained drop oscillations. Water contains organic additives which increase the stability of the drops.

oscillations can be perfectly described by the Rayleigh formula $\omega = (8\sigma/\rho_1 R^3)^{\frac{1}{2}}$ for small-deformation vibrations of a spherical bubble or a drop in a weightless state. Let us note that the authors relate these oscillations to free convection above a heated surface. The existence of similar self-oscillations in 'cool' experiments shows, however, that the reason for this phenomenon should be sought elsewhere. The mechanism of these oscillations seems to be connected with the automatic adjustment

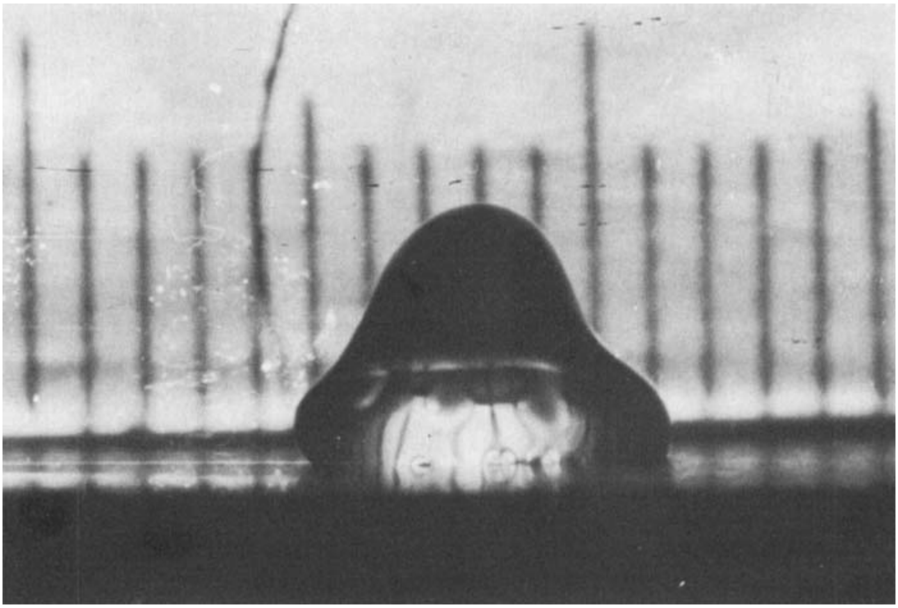

FIGURE 13. Moment of drop capture by the surface when the amplitude of the oscillations has attained the value of the gap thickness.

of the injection (evaporation) intensity to the magnitude of the gap (see also Hall 1974).

On increasing the pressure under the plate the oscillations become very intensive and finally cause the capture of the drop by the surface (figure 13). This creates certain difficulties for the realization of the phenomenon, since it leads to limitations imposed on the permissible blow intensities not only from below, but also from above.†

6.3. *Dependence of h on Δp: critical pressures*

Table 2 presents some data on the dependence of the mean thickness of the gap under the drop h_a on its volume and differential pressure. The value h_a was determined from the electrical capacity C_E of the drop–plate system. The calculations were based on the formula

$$C_E = \int_0^R (2\pi r/h)\,\mathrm{d}r,$$

where $h(r)$ was assumed to be defined by (22). The last line in table 2 represents the values of Δp calculated for given V and h_a in terms of the model of §5.3. Considering the roughness in the determination of the gap thickness, we can conclude that the agreement between the theory and experimental data is satisfactory.

Table 3 and figure 5 present data on the least possible pressure for drop levitation. Curve 4, plotted in terms of the flat-bottom model of §5.3 (taking into account the lateral surface), agrees with experiment better than other theoretical results. According to its sense of a minimum curve, it lies under the experimental points and,

† As for Leidenfrost drops, any section of their contact with the hot wall may evaporate before it attracts and thereby destroys the whole drop. This additional mechanism of stability removes the restriction on the blow intensity from above and results in an interrupting-contact regime typical of the spheroidal state.

considering their spread, can be assumed to be qualitatively satisfactory. It corresponds, in particular, to the rather weak dependence of critical differential pressures on drop sizes (for $R > 0.75a$) that is observed in experiment. The reason for the location of curve 3, obtained in terms of the full formulation of the problem, above the experimental points is not completely clear to us. This may be connected with the development of oscillations on the drop bottom, which make it effectively flat.

7. Conclusion

This paper describes the phenomenon of drop levitation on an air cushion, which is similar to the well-known Leidenfrost phenomenon. It has been shown that this similarity is based on the close analogy between the hydrodynamic mechanisms of drop suspension. One of the mechanisms is connected with the effect of gas- or heat-flow choking under a drop which conditions the threshold character of the phenomena under study. The critical parameter that governs the possibility of levitation is the differential pressure on a porous surface (as does the critical (Leidenfrost) temperature in the thermal problem).

We have developed a mathematical model of drop levitation, applied to the viscous regime of a gas or vapour flow over the gap, that can also be used in the critical range of parameters. As for the Leidenfrost problem, this formulation differs from those developed earlier in excluding *a priori* assumptions about the drop form and in the correct consideration of the choking effect. The flat-bottom approximation has also been considered in which the nonlinear equations for the drop form and the linear equations for the pressure distribution are separated. The simplest form of this approximation (disk model) allows analytical solution. In terms of this model the 'cool' and the thermal problems turn out to be similar and the similarity parameter is determined by the ratio of the gap thickness to the corresponding choking scale. In the exact formulation of the problems this similarity is violated, which is connected with the explicit dependence between the rate of evaporation from the drop bottom (as distinguished from the injection rate) on the local gap thickness.

It leads to various structures of the bottom in the critical range. The bottom of cool drops is convex as $h \rightarrow 0$; the bottom of Leidenfrost drops is concave. At the same time, the form of the lateral surface of the drops is the same: it depends only slightly on loads and can be well calculated in terms of the flat-bottom approximation. The force acting on drops of capillary size is mainly determined by the pressure applied to their underside and can be calculated roughly in terms of the disk model. With decreasing volume, the influence of the lateral surface becomes appreciable. The critical parameters (minimum differential pressure and Leidenfrost temperature) are found to be decreasing functions of the drop volume. Their characteristic values are very small. Thus for water drops of capillary size the critical superheat of the surface amounts to only 0.01 K, if the roughness of the surface is less than the choking scale ($\sim$ 10 μm for $R \sim 1$ cm). However, with increasing roughness, the critical parameters increase sharply.

The experiments performed on disk levitation have confirmed the validity of the ideas about the mechanisms of suspension which underlie the theory developed here. The theory also perfectly describes the form of the lateral surface observed with stationary suspended drops. It offers, in particular, a simple method of determining the volume of the suspended drops in the process of their evaporation. The results obtained from measuring the thickness of the gap and the lowest-possible differential pressures for drop levitation agree only qualitatively with the theoretical predictions. In this case the best results can be obtained in terms of the flat-bottom model with allowance for the lateral surface of the drops.

With increasing injection intensity, self-sustained oscillations of drops develop. Their amplitude may become appreciable, which leads to the capture of a drop by the surface, and so explains why the permissible blow rates are found to be limited from above. The oscillations of drops on an air cushion are quite similar to the oscillations of Leidenfrost drops. This testifies to the hydrodynamic nature of the latter and calls into question the postulated thermoconvective mechanism of these oscillations. This is an example of the use of the phenomenon described as a model of the Leidenfrost phenomenon. Other applications may arise, for instance in studying the interaction between suspended drops.

In conclusion we will note that this research was stimulated by Kutateladze's idea concerning the analogy between boiling and bubbling. The authors are thankful to Academician S. S. Kutateladze for his support and useful discussions.

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